Journal of Mechanical Science and Technology 21 (2007) 855~859

Journal of
Mechanical
Science and
Technology

Novel Adaptive Repetitive Algorithm for Active Vibration Control of a Variable-Speed Rotor

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(Manuscript Received November 16, 2006; Revised January 8, 2007)

Abstract

The paper describes experimental work that demonstrates the use of repetitive control to attenuate radial vibrations of a variable speed-rotor. The experiments were performed on a rotor test rig having a 3-kg rotor supported by journal bearings. The first bending resonance of the rotor shaft (*i.e.* the critical speed) was approximately 50 Hz. The objective was to control the radial response at the rotor midpoint by using an actuator located outside the bearing span. A novel aspect of the controller design is that the length of the control output vector of the repetitive controller was updated as a function of the speed of rotation. The speed of rotation determined the required delay time and the repetitive filter length that approximately matches with the delay time. The results obtained were comparable to those achieved in earlier studies with feedforward compensation methods. The best results were achieved when the frequency of rotation enables an integer ratio between disturbance period and sample rate.

Keywords: Repetitive control; Vibration control; Variable-speed motor

1. Introduction

A number of active vibration control solutions for rotors have been developed in recent years to mitigate the detrimental effects of excitation caused by rotation. The control objective has been to attenuate either the displacement of the rotor, or the transmission of vibration into the surroundings. In previously reported work by the authors of the current paper (Tammi, 2003 ; Daley *el al.,* 2006 for example), the performance of a number of classical vibration control algorithms, such as Convergent Control (Knospe *et al.,* 1997) and the Filtered-X LMS algorithm (see *e.g.* Elliot, 2001) in rotor displacement control have been independently evaluated. These studies have utilised a purpose built experimental rotor rig (Fig. 1) where control is effected using a novel electromagnetic actuator.

The rig comprises a slim shaft, with 10mm diameter, supported by journal bearings that have a span of 360 mm (Fig. 2). The shaft has three disks attached to it making the rotor weigh about 3 kg. The rotor has its first bending resonance mode (*i.e.* the critical speed) at about 50 Hz. The rotor is driven by an electrical motor at one end and an electro-magnetic actuator that produces a radial control force is fixed at the other end. The actuator, modified from a magnetic bearing unit, is run by a separate programmable con-

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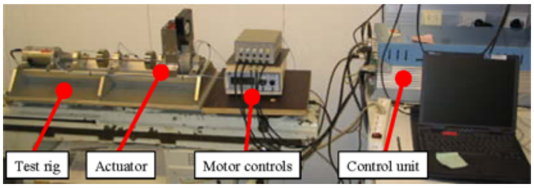

Fig. 1. A general overview of the test environment.

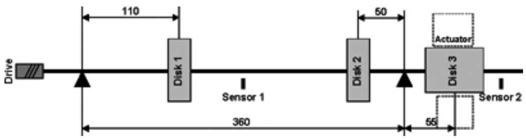

Fig. 2. The layout of the rotor system (the dimensions are in millimetres).

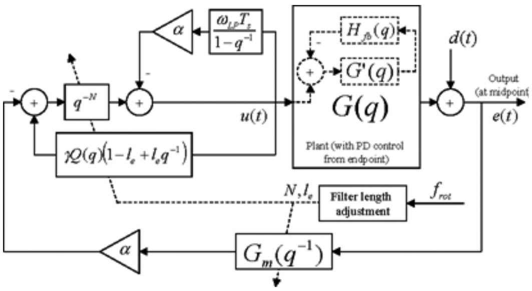

Fig. 3. The schematic overview of the control system applied.

trol unit. The foundation, to which the actuator and the journal bearings are fixed, is assumed to be rigid compared with the flexibility of the rotor.

In the current paper, the rig is utilised to evaluate the application of a novel repetitive control algorithm for rotor control. This algorithm is described in the following section and a benchmark for its performance under transient operation is provided by comparing it with that of convergent control.

2. Control algorithm development

Two cascaded control loops are to be implemented in both orthogonal radial directions (horizontal and vertical directions perpendicular to the rotor shaft). Both loops generate a component of the force command signal to the same actuator. The inner loop is a standard proportional-derivative (PD) feedback loop with a low-pass filter from the endpoint of the rotor. The outer loop consists of a repetitive controller that is adapted as a function of the rotor speed (Fig. 3).

2.1 Proportional-derivative control

The pulse transfer function from the rotor endpoint displacement sensor (*Sensor 2* in Fig. 2) to the actuator force in the corresponding radial direction is

$$H_{fb}(q) = \left(\frac{K_d}{T_S}\left(1 - q^{-1}\right) + K_p \right) \frac{1}{2}\left(q^{-1} + 1\right) \qquad (1)$$

where K_d is the derivative gain, K_p is the proportional gain, $T_S = 0.0001$ s is the sample time, and q is the forward shift operator. The control topology is considered approximately collocated. The main purpose of the PD controllers is to provide sufficient amount of damping at the resonance region. Hence, the derivative gain is relatively large compared to the proportional gain ($K_d = 86$ Ns/m, $K_p = 7$ N/mm). In Fig. 3, $G'(q)$ denotes the transfer function from the actuator force to the rotor endpoint displacement and $G(q)$ stands for the transfer function from the actuator force to the midpoint under the assumption that the PD loops [(*i.e.* $H_{fb}(q)$] are closed.

2.2 Repetitive control with variable time delay

Repetitive controller design has been specifically developed to deal with systems that contain periodic disturbances or reference signals. The principle was first introduced by Inoue *et al.* (1981) and since then a wide variety of approaches have been proposed, (Tomizuka *et al.,* 1989 ; Kempf *et al.,* 1993 ; Medvedev and Hillerström, 1993), for example. The key idea behind the method is to continuously update (or refine) the control output using knowledge of the error in the previous period. The simplest repetitive control algorithm is

$$u(t) = u(t - T) + e(t) \qquad (2)$$

where $u(t)$ are the control outputs, $e(t)$ is the control error and T is the delay time to be set according to the period of the signal to be tracked or compensated. Positive feedback of the delayed control signal leads to infinite feedback gain at the frequencies matching with the period stated by the delay time. Subsequent analysis shows that due to this infinite feedback gain, the loop gain of the controlled system must be positive real (Hätönen, 2004a). This somewhat stringent constraint motivates the use of a more advanced repetitive control law. The starting point for the development of a repetitive controller with adaptive delay time, as required here due to changes in rotational speed, is a gradient based method where the feedback path consists of a truncated FIR filter developed from the plant model. This approach is selected, because it computationally inexpensive and closed-loop stability is guaranteed if the maximum phase error is ±90° with respect to the dynamics of the control path (Hätönen *et al.,* 2004). The update scheme for the gradient-based repetitive controller is

$$u(t) = q^{-N}(\gamma Q(q)u(t) - \alpha G_m(q^{-1})e(t)) \tag{3}$$

where the leak coefficient, $0 < \gamma < 1$, can be used to implement a forgetting factor into the integrator component of the algorithm, and $Q(q)$ is a symmetric filter with zero phase lag

$$Q(q) = (c_P q^{-P} + c_{P-1}q^{-P+1} + \ldots + c_0 + \ldots c_{P-1}q^{P-1} + c_P q^P), P \le N \tag{4}$$

$[c_P \ldots c_0]$ are the FIR coefficients, P is the order of the filter and N is a multiple of the period expressed as an integer number of samples. The coefficient $\alpha > 0$ determines the convergence rate of the algorithm and the update filter $G_m(q^{-1})$ is

$$G_m(q^{-1}) = (a_M q^M + a_{M-1}q^{M-1} + \ldots + a_1 q + a_0), M \le N \tag{5}$$

where $[a_M \ldots a_0]$ are the FIR coefficients, determined from the time reversed impulse response of the plant $G(q)$, and M is the order of the filter. Even though the plant model and the Q-filter are non-causal filters, the overall algorithm is causal, if the filter orders M and N do not exceed the delay length N, as q^{-N} is the common factor in the control law in Eq. (3). Maintaining the causality restricts the length of the FIR approximation and thus its accuracy. The FIR approximation may also cause destabilising effects but this can be avoided using windowing techniques (Chen and Longman 2002). In the novel algorithm developed in this section, the delay time is selected adaptively according to the rotor speed measurement. Firstly, the integer number of samples (N) required is determined and this is always rounded downwards. Secondly, the fact that the required delay time does not necessarily meet with the integer number of samples is taken into account using interpolation. For the interpolation, a length error ($0 \le l_e < 1$) is defined that describes the relative error due to rounding. The number of samples and the length error are defined as

$$N = \text{floor}\left(\frac{1}{f_{rot}T_S}\right), \quad l_e = \frac{1}{f_{rot}T_S} - N \tag{6}$$

where N is the number of samples, f_{rot} is the measured speed of rotation in revolutions per second (rps), and l_e is the length error. Both these parameters are then used in the control law in Eq. (7).

The repetitive control method, being integrative, provides high feedback gain at zero frequency (DC). The band-pass filter was, however, impossible to implement in the Q-filter due to technical restrictions. Having a FIR filter with low DC gain and unity amplification at frequencies starting about 20 Hz was not realisable with the chosen filter lengths. This problem was solved by implementing a separate DC removal integrator in the control law. The control law with interpolation and DC removal functions is

$$\begin{aligned} u(t) &= \gamma q^{-N} Q(q)\left((1-l_e) + l_e q^{-1}\right)u(t) \\ &\quad - \alpha q^{-N} G_m(q^{-1})e(t) - \alpha\left(\frac{\omega_{LP}T_S}{1-q^{-1}}\right)u(t) \end{aligned} \tag{7}$$

where ω_{LP} is the integrator gain, used for the adjustment of low-pass corner frequency (ω_{LP} = 6.2 rad/s). The interpolation and the DC removal integrator modify the phase of the feedback loops and have some destabilising effects on the algorithm. This feature contradicts the original idea having zero-phase feedback loops to ensure stability (Tomizuka *et al.,* 1989). Stability analysis is outside the scope of this paper and is considered in a later publication (Tammi, 2007), however, it can be stated that a low-pass-type Q-filter was found to be essential in maintaining stability at high frequencies.

3. Experimental results

The results of applying algorithm (7) to the rotor rig (Fig.1) are presented in this section. The transfer functions of the plant, the system models of two different orders, and the Q-filter are shown in Fig. 4.

The experimental results at steady-state conditions with different constant speeds have previously reported in Tammi *et al.* (2006), here, variable speed is considered. Figure 5 shows a comparison of the radial displacement responses at the rotor midpoint during a run-up with a rate of 4000 rpm/min (1.1 rps/s). The response with feedback control alone is shown as a reference. This response was measured up to 43 rps only in order to avoid large rotor amplitudes at the resonance region (the steady-state peak amplitude is estimated to be about 150 μm at the critical speed). The response of the rotor with feed-back and the repetitive control operating together fluctuated as a function of the rotor speed. The response reduced when the rotor speed enabled an integer ratio between the disturbance period and the sample rate (*i.e.* $l_e \approx 0$).

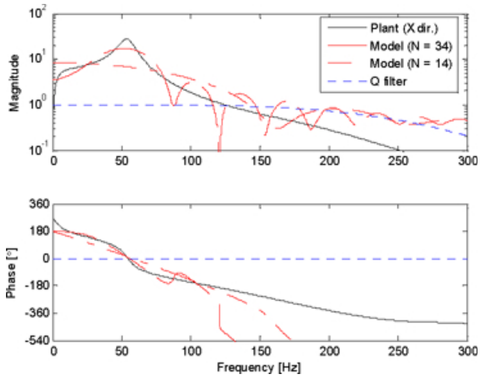

Fig. 4. The frequency responses of the plant, the plant models of two different orders and the Q-filter.

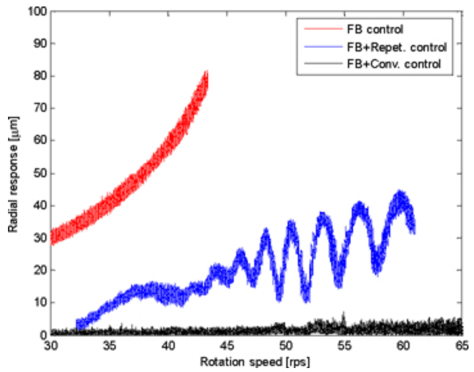

Fig. 5. The radial responses of the rotor midpoint when running with feedback control only, with repetitive control and with convergent control.

The implementation of the repetitive control method provided a maximum attenuation of 12 dB at the midpoint response. This attenuation enabled the rotor to run in the super-critical region. The convergence of the algorithm became more rapid at speeds between 47 rps and 62 rps as the gain norm of the feedback path increased.

A benchmark response obtained with the feedforward convergent control method (Daley *et al.*, 2006) is also shown in Fig. 5. Although this produces superior reduction at all speeds, the repetitive controller has the advantage of targeting both the rotational frequency and all harmonics simultaneously.

4. Conclusions

A gradient-based repetitive control law has been modified for vibration control of a variable-speed rotor by utilising an adaptive time delay. Due to issues with modelling uncertainty, interpolation between successive samples, and the implementation of a DC removal filter, the use of a Q-filter was required in order to avoid high-frequency control actions. The achievable disturbance attenuation was highly dependent on the rotation speed; the largest attenuation was achieved when the rotor speed enabled an integer ratio between the disturbance period and the sample rate. The steady-state performance of the repetitive control method was close to that obtained previously with feedforward control methods when the delay time matched with the rotation frequency, otherwise, the performance of repetitive control was worse. The performance difference between repetitive control and feedforward control was larger for a run-up condition than for a steady-state condition. It is suspected that this feature may be caused by a longer convergence time following transient excitation for the repetitive control algorithm compared to the feedforward methods.

The experimental results suggested that a feedforward control method may be a more attractive active vibration control solution for variable-speed rotors than the presented repetitive control method. However, the presented work did not make full justice for the repetitive control method's ability to reject any periodic disturbance matching with the delay time (limited by the Q-filter) in the context of active control of vibrations. In certain rotating machines, the frequentcies of the most significant excitations may not be predictable.

Future work will develop a stability analysis with respect to the interpolation and the DC removal functionality. The algorithm presented in this paper will also be tested on an electrical machine, where the objective is to control internal radial forces of the machine in order to reduce rotor vibrations.

Acknowledgements

The work was funded by VTT and by the Academy of Finland (funding decision #110553).

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